

Phenomenology of a new minimal supersymmetric extension of the standard model

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We study the phenomenology of a new minimally extended supersymmetric standard model (nMSSM) where a gauge singlet superfield is added to the MSSM spectrum. The superpotential of this model contains no dimensionful parameters, thus solving the μ problem of the MSSM. A global discrete R symmetry, forbidding the cubic singlet self-interaction, imposed on the complete theory, guarantees its stability with respect to generated higher-order tadpoles of the singlet and solves both the domain wall and Peccei-Quinn axion problems. We give the free parameters of the model and display some general constraints on them. Particular attention is devoted to the neutralino sector where a (quasipure) singlino appears to be *always* the LSP of the model, leading to additional cascades, involving the NLSP \rightarrow LSP transition, compared with the MSSM. We then present the upper bounds on the masses of the lightest and next-to-lightest—when the lightest is an invisible singlet— CP -even Higgs bosons, including the full one-loop and dominant two-loop corrections. These bounds are found to be much higher than the equivalent ones in the MSSM. Finally, we discuss some phenomenological implications for the Higgs sector of the nMSSM in Higgs boson production at future hadron colliders.

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I. INTRODUCTION

Supersymmetry provides a well-defined framework for the study of physics beyond the standard model (SM). Its main motivation has been the special properties of supersymmetric (SUSY) theories with respect to the hierarchy problem. In addition, the low-energy data support unification of the gauge couplings in the SUSY case, in contrast with what happens in the SM scenario. Another interesting feature of SUSY models is that the breaking of the electroweak (EW) symmetry can be radiatively triggered by the largeness of the top quark mass [1]. The minimal supersymmetric standard model (MSSM) [2] is defined by promoting each standard field into a superfield, doubling the Higgs fields and imposing R -parity conservation. Because of the nonobservation of superpartners of the standard particles, supersymmetry has to be broken at a scale M_{SUSY} not larger than $O(\text{TeV})$, so that it still provides a natural solution to the hierarchy problem. Unfortunately, a phenomenologically acceptable realization of EW symmetry breaking in the MSSM requires the presence of the so-called μ term, a direct SUSY mass term for the Higgs fields, with values of the (theoretically arbitrary) parameter μ close to M_{SUSY} or M_W , when its natural value would be either 0 or the Planck mass, M_P . Of course, there exist explanations for an $O(M_W)$ value of the μ term, alas, all in extended settings [3].

The more or less straightforward solution to the μ -problem is to promote the μ parameter into a field whose vacuum expectation value (VEV) is determined, as the other scalar field VEV's, from the minimization of the scalar potential along the new direction [4–7]. Naturally, it is expected to fall in the range of the other VEV's, i.e., of order $O(M_{\text{SUSY}})$. Such a superfield has to be a singlet under the SM gauge group. In order to avoid introducing new scales

into the model one should stick to dimensionless couplings at the renormalizable level. This can be achieved by imposing a \mathbb{Z}_3 symmetry on the renormalizable part of the superpotential. The resulting model, the next-to-minimal supersymmetric standard model (NMSSM), has the following superpotential:

$$W = \lambda S H_1 H_2 + \frac{\kappa}{3} S^3 + \dots, \quad (1)$$

where the ellipsis stands for the usual quark and lepton Yukawa couplings [see Eq. (2)]. The \mathbb{Z}_3 symmetry is spontaneously broken at the EW scale when the Higgs fields get a nonzero VEV. It is well known, however, that the spontaneous breaking of such a discrete symmetry results in disastrous *cosmological domain walls*, unless this symmetry is explicitly broken by the nonrenormalizable sector of the theory. Domain walls can be tolerated if there is a discrete-symmetry-violating contribution to the scalar potential larger than the scale $O(1 \text{ MeV})$ set by nucleosynthesis [8]. Heavy fields interacting with the standard light fields generate in the effective low-energy theory an infinite set of nonrenormalizable operators of the light fields scaled by powers of the characteristic mass scale of the heavy sector (M_P , M_{GUT} , \dots). These terms appear either as D terms in the Kähler potential or as F terms in the superpotential. It is known, however, that gauge singlet superfields *do not obey decoupling* [3,9], so that, when supersymmetry is either spontaneously or softly broken, in addition to the suppressed nonrenormalizable terms, they can in general give rise to a large tadpole term in the potential proportional to the heavy scale: $M_{\text{SUSY}}^2 M_P (S + S^*)$. Technically, the tadpole is generated through higher-order loop diagrams in which the nonrenormalizable interactions participate as vertices together

with the renormalizable ones. A discrete global symmetry similar to the one discussed above would forbid this term but would lead to the appearance of disastrous domain walls upon its unavoidable spontaneous breakdown. The generated large tadpole reintroduces the hierarchy problem, since due to its presence the singlet VEV gets a value $\langle S \rangle^2 \sim M_{\text{SUSY}} M_P$. It appears that $N=1$ supergravity, spontaneously broken by a set of hidden sector fields, is the natural setting to study the generation of the destabilizing tadpoles. A thorough analysis carried out in Ref. [10] shows that the only harmful nonrenormalizable interactions are either even superpotential terms or odd Kähler potential ones. In addition, operators with more than six powers of the cutoff in the denominator are harmless. Finally, a tadpole diagram is divergent only if it contains an *odd* number of “dangerous” vertices.

The solution of the μ problem in the framework of the NMSSM could be rendered a viable one if the destabilization problem were circumvented. What is needed is a suitable symmetry that forbids the dangerous nonrenormalizable terms and allows only for tadpoles of order $M_{\text{SUSY}}^3(S+S^*)$. This symmetry should at the same time allow for a large enough Z_3 -breaking term in the scalar potential in order to destroy the unwanted domain walls [11].

An alternative approach is to impose a symmetry which, although it does not forbid the dangerous nonrenormalizable terms, only allows for higher-order tadpole graphs that give a n -loop-suppressed term $[1/(16\pi^2)^n] M_{\text{SUSY}}^2 M_P (S+S^*)$. A case of particular interest is when the cubic self-interaction for the singlet in Eq. (1) is forbidden by the symmetry. Actually, it should be noted that if the underlying theory is a grand unified theory (GUT), although a candidate for the singlet exists, a cubic term does not arise.¹ On the other hand, this case is truly minimal in the sense that, apart from promoting the μ parameter into a field, no new renormalizable terms appear in the superpotential. Of course, a substitute is needed for the twofold role played by the cubic term, namely, its contribution to the mechanism generating the VEV of S through the soft SUSY breaking terms and the breaking of the Peccei-Quinn symmetry present when $\kappa=0$. This role can be played by the tadpole. (Note that this is not included in the $\kappa \rightarrow 0$ limit of the existing NMSSM analyses, which up to now have ignored the tadpole term [5,6,12].) Recently, a viable solution along these lines was proposed based on discrete R symmetries [13]. The renormalizable superpotential for this new minimal supersymmetric extension of the standard model (nMSSM) is given by

$$W = \lambda S H_1 H_2 + Y_u Q U^c H_1 + Y_d Q D^c H_2 + Y_e L E^c H_2. \quad (2)$$

Apart from the usual Baryon and Lepton number, it pos-

sesses two additional global continuous symmetries, namely, an anomalous Peccei-Quinn symmetry $U(1)_{PQ}$ with charges

$$\begin{aligned} Q(-1), \quad U^c(0), \quad D^c(0), \quad L(-1), \\ E^c(0), \quad H_1(1), \quad H_2(1), \quad S(-2) \end{aligned} \quad (3)$$

and a nonanomalous R -symmetry $U(1)_R$ with charges

$$\begin{aligned} Q(1), \quad U^c(1), \quad D^c(1), \quad L(1), \quad E^c(1), \\ H_1(0), \quad H_2(0), \quad S(2). \end{aligned} \quad (4)$$

One of the solutions worked out consists in imposing the discrete sub-symmetry Z_{5R} of the $U(1)_{R'}$ combination $R' = 3R + PQ$ on the complete theory, including nonrenormalizable operators. The charges under Z_{5R} are

$$\begin{aligned} (H_1, H_2) &\rightarrow \alpha(H_1, H_2), \\ (Q, L) &\rightarrow \alpha^2(Q, L), \\ (U^c, D^c, E^c) &\rightarrow \alpha^3(U^c, D^c, E^c), \\ S &\rightarrow \alpha^4 S, \\ \mathcal{W} &\rightarrow \alpha \mathcal{W}, \end{aligned} \quad (5)$$

where $\alpha = e^{2i\pi/5}$. An adequately suppressed linear term is generated at six-loop level by combining the nonrenormalizable Kähler potential terms $\lambda_1 S^2 H_1 H_2 / M_P^2 + \text{H.c.}$ and $\lambda_2 S (H_1 H_2)^3 / M_P^5 + \text{H.c.}$ with the renormalizable superpotential term $\lambda S H_1 H_2$:

$$V_{\text{tadpole}} \sim \frac{1}{(16\pi^2)^6} \lambda_1 \lambda_2 \lambda^4 M_{\text{SUSY}}^2 M_P (S + S^*). \quad (6)$$

This tadpole has the desired order of magnitude $O(M_{\text{SUSY}})$ if $\lambda_1 \lambda_2 \lambda^4 \sim 10^{-3}$.

The goal of this paper is the phenomenological study of this nMSSM, where a gauge singlet superfield is added to the MSSM spectrum and a global Z_{5R} symmetry is imposed on the complete theory, resulting in the superpotential of Eq. (2) and the tadpole term of Eq. (6). In Sec. II we review the general properties of the parameter space of the model. Phenomenological aspects of the nMSSM are addressed in Sec. III. The neutralino sector, including the (quasi-pure) singlino, is studied in some detail. Also bounds on CP -even Higgs masses versus their couplings to gauge bosons are displayed along with Higgs production cross sections at future hadron colliders. Section IV contains our main conclusions.

II. MODEL SETUP

The tree-level Higgs scalar potential, namely, the potential which contains the scalar fields $H_1 = (H_1^0, H_1^-)$, $H_2 = (H_2^+, H_2^0)$ and S , has the form

$$V^{(0)} = V_F + V_D + V_{\text{soft}} + V_{\text{tadpole}}, \quad (7)$$

¹In E_6 , for example, matter and Higgs fields are contained in the **27** representation together with a singlet. Although the standard trilinear singlet Higgs field term is present in the **27**³ coupling, no singlet cubic term arises. The same is true for E_6 embeddings of $SO(10)$ and $SU(5)$.

$$V_F = |\lambda|^2 [(|H_1|^2 + |H_2|^2) |S|^2 + |H_1|^2 |H_2|^2] - |\lambda|^2 (H_1^{0*} H_2^{0*} H_1^- H_2^+ + \text{H.c.}), \quad (8)$$

$$V_D = \frac{g_1^2 + g_2^2}{8} [|H_1|^2 - |H_2|^2]^2 + \frac{g_2^2}{2} |H_1^\dagger H_2|^2, \quad (9)$$

$$V_{\text{soft}} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_S^2 |S|^2 + (\lambda A_\lambda S H_1 H_2 + \text{H.c.}). \quad (10)$$

In what follows, we shall assume a phenomenological point of view and write the generated tadpole as

$$V_{\text{tadpole}} \equiv \xi^3 (S + S^*), \quad (11)$$

where ξ is treated as a free parameter.

In order to obtain the correct upper limits on the Higgs boson masses (see Sec. III B) radiative corrections to the tree-level potential have to be considered. Let us introduce a scale $Q \sim M_{\text{SUSY}}$ and assume that quantum corrections involving momenta $p^2 \gtrsim Q^2$ have been evaluated, e.g., by the integration of the renormalization group equations (RGEs) of the parameters from initial values at the GUT scale down to the scale Q . One is then left with the computation of quantum corrections involving momenta $p^2 \lesssim Q^2$. The effective potential V_{eff} can be developed in powers of \hbar or loops as

$$V_{\text{eff}} = V^{(0)} + V^{(1)} + V^{(2)} + \dots \quad (12)$$

The tree-level potential $V^{(0)}$ is given by Eq. (7). The one-loop corrections to the effective potential read as

$$V^{(1)} = \frac{1}{64\pi^2} \text{STr} M^4 \left[\ln \left(\frac{M^2}{Q^2} \right) - \frac{3}{2} \right], \quad (13)$$

where M^2 is the field dependent squared mass matrix (in our analysis, we take only top-quark–top-squark loops into account). Next, we consider the dominant two-loop corrections. These will be numerically important only for large SUSY breaking terms compared to the Higgs VEV's h_i , hence we can expand in powers of h_i . Since the terms quadratic in h_i can be absorbed into the tree-level soft terms, we just consider the quartic terms, and here only those which are proportional to large couplings: terms $\sim \alpha_s h_i^4$ and $\sim h_i^6$. Finally, taking only leading logarithms (LLs) into account, the expression for $V^{(2)}$ reads

$$V_{\text{LL}}^{(2)} = 3 \left(\frac{h_t^2}{16\pi^2} \right)^2 h_t^4 \left(32\pi\alpha_s - \frac{3}{2} h_t^2 \right) t^2, \quad (14)$$

where $t \equiv \ln(Q^2/m_t^2)$, m_t being the top-quark mass. One-loop corrections to the tree-level relations between bare parameters and physical observables, once reinserted in the one-loop effective potential, also appear as two-loop effects. These are corrections to the kinetic terms of the Higgs bosons, which lead to a wave function renormalization factor Z_{H_2} in front of the $D_\mu H_2 D^\mu H_2$ term with, to order h_t^2

$$Z_{H_2} = 1 + 3 \frac{h_t^2}{16\pi^2} t \quad (15)$$

and corrections to the top-quark Yukawa coupling with, to orders h_t^2 , α_s

$$h_t(m_t) = h_t(Q) \left[1 + \frac{1}{32\pi^2} \left(32\pi\alpha_s - \frac{9}{2} h_t^2 \right) t \right]. \quad (16)$$

In general, the parameters λ , A_λ , and ξ could be complex. However, by redefining the fields H_2 (or H_1) and S , one can always get—without loss of generality—that $\lambda A_\lambda, \xi^3 \in \mathbb{R}$. Note that in the NMSSM with the cubic singlet superpotential term $\frac{1}{3} \kappa S^3$ one has to further *assume* that the combination $\lambda \kappa^*$ (or, equivalently, A_λ/A_κ) is real [5,6]. By $SU(2)_L \times U(1)_Y$ gauge invariance one can get rid of the phase of H_1 , by taking $\langle H_1^- \rangle = 0$ and $h_1 \equiv \langle H_1^0 \rangle \in \mathbb{R}^+$. One can then show that the condition for a local minimum with $\langle H_2^+ \rangle = 0$ is equivalent to a positive mass squared for the charged Higgs boson. It has been proven that a sufficient condition is $\lambda < g_2$ [14] which, as we shall see below, is always verified in the universal case. By taking $h_2 \equiv \langle H_2^0 \rangle = \rho_2 e^{i\phi_2}$, $s \equiv \langle S \rangle = \rho_0 e^{i\phi_0}$ and minimizing the complete (two-loop) effective potential with respect to ϕ_0 and ϕ_2 , we find that there is one and only one global vacuum for which the two phases relax to zero, i.e., $\phi_0 = \phi_2 = 0$. This implies that there is no spontaneous CP violation. Therefore one can choose $h_1 \in \mathbb{R}^+$ and $h_2, s \in \mathbb{R}$. This result distinguishes the nMSSM from the usual NMSSM where loop corrections can generate spontaneous CP violation [15].

The soft terms of the model can be constrained by requiring universality at the GUT scale. The independent parameters of the model are then a universal gaugino mass $M_{1/2}$ (always positive in our convention), a universal mass for the scalars m_0^2 , a universal trilinear coupling A_0 (either positive or negative), the (positive) Yukawa coupling λ_0 at the scale M_{GUT} and the tadpole coefficient ξ . The (well-known) value of the Z-boson mass fixes one of these parameters with respect to the others, so that we end up with four free parameters at the GUT scale, i.e., as many as in the MSSM with universal soft terms. In principle, one could choose the same set of free parameters as in the MSSM, i.e., $M_{1/2}$, m_0^2 , A_0 , and $\tan\beta (\equiv h_2/h_1)$, with λ , s , and ξ being determined by the three minimization equations, demanding also radiative electroweak symmetry breaking [1]. However, this appears to be a nontrivial issue, as λ also influences the running of the RGEs of the soft parameters between the GUT and the EW scale. In other terms, one would need a lot of fine tuning of the dimensionful A_0 in order to get a dimensionless parameter λ of the desired value at the EW scale. Therefore, in the case of universality, we conveniently adopt in our numerical analysis the following input parameters: $m_0^2/M_{1/2}$, $A_0/M_{1/2}$, $\xi/M_{1/2}$, and λ_0 ($\tan\beta$ and s being calculated from the minimization of the potential and the overall scale $M_{1/2}$ fixed by M_Z).

If one requires the absence of a Landau singularity for λ below the GUT scale, one obtains an upper bound on λ at the

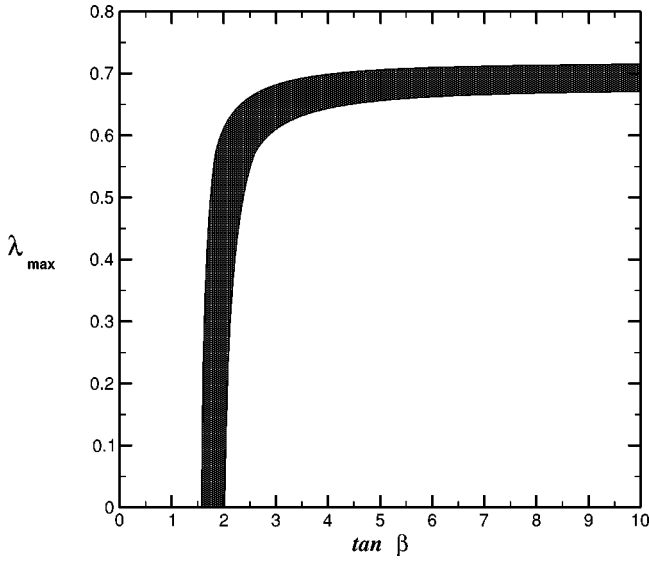


FIG. 1. Upper bound on λ as a function of $\tan \beta$ for $m_t^{\text{pole}} = 173.8 \pm 5.2$ GeV [18]. The width of the curve is due to the uncertainty on m_t^{pole} .

EW scale. This upper bound depends on the value of the top-quark Yukawa coupling h_t , i.e., on $\tan \beta$ (cf. Fig. 1). Requiring furthermore universality at the GUT scale, one ends up with a more restrictive constraint $\lambda \lesssim 0.3$, higher values leading to unphysical global minima of the effective potential.

Let us now briefly address the problem of charge and color breaking (CCB) minima. The most dangerous CCB direction involves the trilinear coupling $h_e A_e E_{R,1} L_1 H_1$ where h_e denotes the electron Yukawa coupling ($\sim 10^{-5}$), $E_{R,1}$ is the right-handed selectron, L_1 the left-handed slepton doublet of the first generation, and H_1 the corresponding Higgs doublet. From the absence of a non-trivial minimum of the scalar potential in the D -flat direction $|E_{R,1}| = |L_1| = |H_2|$, the following inequality among the soft SUSY breaking terms can be derived [4]:

$$A_e^2 < 3(m_E^2 + m_L^2 + m_1^2), \quad (17)$$

where m_E^2 , m_L^2 , and m_1^2 are the soft SUSY breaking mass terms associated with the three fields above. If the inequality (17) is violated, the fields develop VEV's of $O(A_e/h_e)$ and the depth of the minimum is of $O(A_e^4/h_e^2)$. Accordingly, Eq. (17) has to be imposed at a scale $Q \sim A_e/h_e \sim 10^7$ GeV. Assuming universal soft terms at the GUT scale, Eq. (17) then becomes [6]

$$(A_0 - 0.5M_{1/2})^2 < 9m_0^2 + 2.67M_{1/2}^2. \quad (18)$$

So-called unbounded from below (UFB) directions (which actually never occurs in the universal case) can also be considered. Assuming universality for the soft terms, the absence of a global minimum in these directions typically implies [16,17]

$$\frac{m_0}{M_{1/2}} \gtrsim 1. \quad (19)$$

However, the tunnelling rate from the standard EW minimum to a UFB one is in general quite small [16], so that this constraint can be avoided if one is ready to assume that the standard EW vacuum is metastable.

III. PHENOMENOLOGICAL ASPECTS OF THE nMSSM

A. Singlino LSP and additional cascades

The nMSSM contains additional gauge singlet states in the Higgs sector (one neutral CP -even and one CP -odd state) and in the neutralino sector (a two component Weyl fermion). These states are mixed with the corresponding ones of the MSSM, and the physical states have to be obtained from the diagonalization of the mass matrices in each sector. In the basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S})$ the (symmetric) neutralino mass matrix reads as

$$\mathcal{M}^0 = \begin{pmatrix} M_1 & 0 & -g_1 h_1 / \sqrt{2} & g_1 h_2 / \sqrt{2} & 0 \\ & M_2 & g_2 h_1 / \sqrt{2} & -g_2 h_2 / \sqrt{2} & 0 \\ & & 0 & \lambda s & \lambda h_2 \\ & & & 0 & \lambda h_1 \\ & & & & 0 \end{pmatrix}. \quad (20)$$

Note that the diagonal singlino (\tilde{S}) mass term is zero. Furthermore, the singlino mixings (with Higgsinos) are proportional to λ , which, as remarked earlier, turns out to be quite small, especially in the universality scenario. Consequently, the singlino state appears to be an almost pure singlet state with a very small mass, so that it is always the lightest supersymmetric particle (LSP) of the model. Actually, in the universal case, we find $\text{few MeV} \lesssim m_{\tilde{S}} \lesssim 3$ GeV, with a singlet component $\gtrsim 99\%$. This state has only small couplings to the gauge bosons and to the other sparticles. (We have explicitly checked that its contribution to the invisible Z-boson width is < 4.2 MeV [18].) Thus, the production cross sections of the singlino are small and it seems to be nearly impossible to observe this particle in any experiment, this rendering the nMSSM apparently similar to the ordinary MSSM. However, as the singlino is the LSP of the model, it will appear at the end of all sparticle decay chains, giving rise to additional cascades compared with the MSSM signals. Such additional cascades are common to many supersymmetric extensions involving singlets [12,19]. It should be noticed, however, that unlike in the NMSSM, where the singlino LSP scenario requires strong constraints on the parameter space (i.e., $M_{1/2} \gg m_0, A_0$) [12], the singlino is *always* the LSP in the nMSSM.

In addition, by assuming universality at the GUT scale, we find that the next-to-lightest supersymmetric particle (NLSP) is always the second lightest neutralino, which turns out to be a quasipure bino (\tilde{B}). Depending on the region of the parameter space under scrutiny, the following channels

can play a role in the NLSP \rightarrow LSP cascade: $\tilde{B} \rightarrow \tilde{S} \nu \bar{\nu}$ (sneutrino/Z exchange) giving an invisible cascade; $\tilde{B} \rightarrow \tilde{S} l^+ l^-$ (slepton/Z exchange) where the leptons could be mainly τ 's, the stau being lighter than the other sleptons; $\tilde{B} \rightarrow \tilde{S} q \bar{q}$ (squark/Z exchange) the branching ratio being quite small ($\lesssim 10\%$), as the squarks are usually heavy; $\tilde{B} \rightarrow \tilde{S} Z$ if the \tilde{B} is heavy enough; $\tilde{B} \rightarrow \tilde{S} S$ where S is a light quasi-pure singlet Higgs boson, decaying to $b\bar{b}$ or $\tau\bar{\tau}$ depending on its mass; $\tilde{B} \rightarrow \tilde{S} \gamma$ through loops. The properties of these cascades have been analyzed in detail in Ref. [12] for the case of the NMSSM and most of the results can equally apply to the case of the nMSSM. As for experimental searches, high multiplicity events have been under investigation already in the context of models with gauge mediated supersymmetry

breaking [20] or with R -parity violation [21]. In principle, small λ 's ($\lesssim 10^{-4}$) could give rise to a delayed NLSP \rightarrow LSP transition, i.e., a displaced neutral vertex [12,19]. However, such values of λ are disfavored if one wants the tadpole term of Eq. (6) to be large enough, so that displaced neutral vertices are not expected as typical signatures of the nMSSM.

B. Higgs couplings and mass bounds

The Higgs sector of the nMSSM consists of three CP -even neutral states, denoted by S_i with masses $m_1 < m_2 < m_3$, plus two CP -odd neutral states, labeled as P_i with masses $m'_1 < m'_2$. The tree-level mass matrix for the CP -even states in the basis $(\text{Re } H_1^0, \text{Re } H_2^0, \text{Re } S)$ reads

$$\mathcal{M}_S^2 = \begin{pmatrix} g^2 h_1^2 - \lambda s A_\lambda \tan \beta & (2\lambda^2 - g^2) h_1 h_2 + \lambda s A_\lambda & \lambda (2\lambda s h_1 + A_\lambda h_2) \\ g^2 h_2^2 - \lambda s A_\lambda \cot \beta & \lambda (2\lambda s h_2 + A_\lambda h_1) & \\ -\lambda^2 A_\lambda \frac{h_1 h_2}{\lambda s} - \lambda \frac{\xi^3}{\lambda s} & & \end{pmatrix}, \quad (21)$$

where $g^2 = (g_1^2 + g_2^2)/2$. In the reminder of this section, we study the upper bounds on the lightest CP -even states with general soft SUSY breaking terms, in a nonuniversal scenario. By taking into account the full one-loop and the dominant two-loop top-quark–top-squark corrections displayed in Sec. II, and assuming $h_i \ll M_{\text{SUSY}}$, one obtains the following upper limit on the lightest CP -even Higgs boson mass:

$$\begin{aligned} m_1^2 \leq & M_Z^2 \left(\cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right) \left(1 - \frac{3h_t^2}{8\pi^2} t \right) \\ & + \frac{3h_t^2(m_t)}{4\pi^2} m_t^2(m_t) \sin^2 \beta \left[\frac{1}{2} \tilde{X}_t + t + \frac{1}{16\pi^2} \right. \\ & \times \left. \left(\frac{3}{2} h_t^2 - 32\pi\alpha_s \right) (\tilde{X}_t + t)t \right], \end{aligned} \quad (22)$$

where

$$\tilde{X}_t \equiv 2 \frac{\tilde{A}_t^2}{M_{\text{SUSY}}^2} \left(1 - \frac{\tilde{A}_t^2}{12M_{\text{SUSY}}^2} \right), \quad (23)$$

$$\tilde{A}_t \equiv A_t - \lambda s \cot \beta, \quad (24)$$

A_t being the top trilinear soft term.

The only difference between the MSSM bound [22] and Eq. (22) is the “tree-level” contribution $\sim \lambda^2 \sin^2 2\beta$. This term is important for moderate values of $\tan \beta$. Hence, the maximum of the lightest Higgs mass in the nMSSM is not obtained for large $\tan \beta$ values, as in the MSSM, rather for

moderate ones (see Fig. 2). In contrast, the radiative corrections are identical in the nMSSM and in the MSSM. In particular, the linear dependence in \tilde{X}_t is the same in both models. Hence, from Eq. (23), the upper bound on m_1^2 is maximized for $\tilde{X}_t = 6$ (corresponding to $\tilde{A}_t = \sqrt{6} M_{\text{SUSY}}$, the “maximal mixing” case) and minimized for $\tilde{X}_t = 0$ (corresponding to $\tilde{A}_t = 0$, the “no mixing” case).

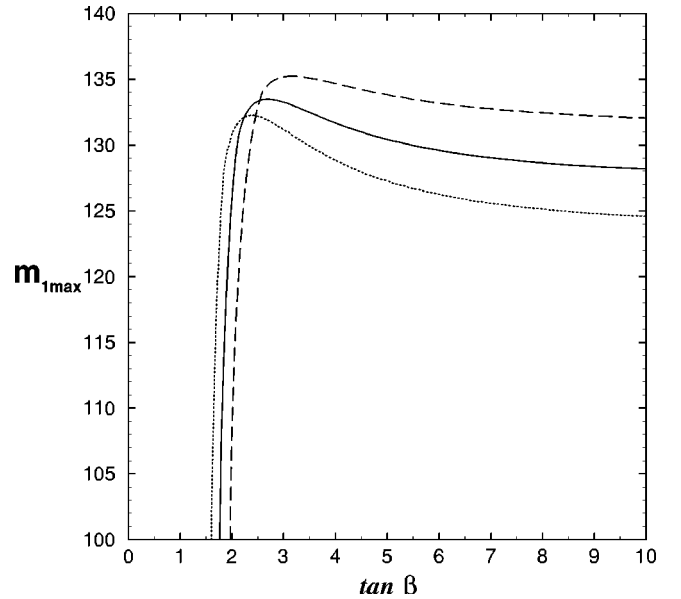


FIG. 2. Upper bound on m_1 [GeV] versus $\tan \beta$ for $m_t^{\text{pole}} = 173.8 \pm 5.2$ GeV (straight, dashed, dotted line, respectively), and $M_{\text{SUSY}} \leq 1$ TeV.

However, the upper limit on m_1 is not necessarily physically relevant, since the coupling of the lightest CP -even Higgs boson to the Z -boson can be very small. Actually, this phenomenon can also appear in the MSSM, if $\sin^2(\beta-\alpha)$ is small. In this case though, the CP -odd Higgs boson A is necessarily light ($m_A \sim m_h < M_Z$ at tree-level) and the process $e^+e^- \rightarrow Z \rightarrow hA$ can be used to cover this region of the MSSM parameter space. In the nMSSM, a small gauge boson coupling of the lightest Higgs S_1 is usually related to a large singlet component, in which case no (strongly coupled) light CP -odd Higgs boson is available. Hence, Higgs searches in the nMSSM have to possibly rely on the search for the second lightest Higgs scalar S_2 .

Let us define the reduced coupling R_i as the square of the ZZS_i coupling divided by the corresponding standard model Higgs- Z boson coupling

$$R_i = (S_{i1} \cos \beta + S_{i2} \sin \beta)^2, \quad (25)$$

where S_{i1}, S_{i2} are the H_1, H_2 components of the CP -even Higgs boson S_i , respectively. Evidently, we have $0 \leq R_i \leq 1$ and unitarity implies

$$\sum_{i=1}^3 R_i = 1. \quad (26)$$

We are interested in upper bounds on the two lightest CP -even Higgs bosons $S_{1,2}$. These are obtained in the limit where the third Higgs boson, S_3 , is heavy and decoupled, i.e., $R_3 \sim 0$ (this scenario is similar the so-called decoupling limit in the MSSM: the upper bound on the lightest Higgs boson h is saturated when the second Higgs boson H is heavy and decouples from the gauge bosons). In this limit, we have $R_1 + R_2 \approx 1$.

In the regime $R_1 \geq 1/2$, experiments will evidently first discover the lightest Higgs boson (with $m_1 \leq 133.5$ GeV for $m_t^{\text{pole}} = 173.8$ GeV and $M_{\text{SUSY}} = 1$ TeV). The “worst case scenario” in this regime corresponds to $m_1 \approx 133.5$ GeV and $R_1 \approx 1/2$: the presence of a Higgs boson with these properties has to be excluded in order to test this part of the parameter space of the nMSSM.

In the regime $R_1 < 1/2$ (i.e., $1/2 < R_2 \leq 1$) the lightest Higgs boson may escape detection because of its small coupling to gauge bosons, and it may be easier to look for the second lightest Higgs S_2 . In Fig. 3 we show the upper limit on m_2 as a function of R_2 as a thin straight line. For $R_2 \rightarrow 1$ (i.e., $R_1 \rightarrow 0$), the upper limit on m_2 is actually given by the previous upper limit on m_1 , even if the corresponding Higgs boson is the second lightest one. For $R_2 \rightarrow 1/2$, on the other hand, m_2 can be as large as 190 GeV. However, one finds that the upper limit on m_2 is saturated when the mass m_1 of the lightest Higgs boson tends to 0. Clearly, one has to take into account the constraints from Higgs boson searches which apply to reduced couplings $R < 1/2$, i.e., lower limits on m_1 as a function of $R_1 \approx 1 - R_2$, in order to obtain realistic upper limits on m_2 versus R_2 . The dotted curves in Fig. 3 show the upper limit on m_2 as a function of R_2 for different

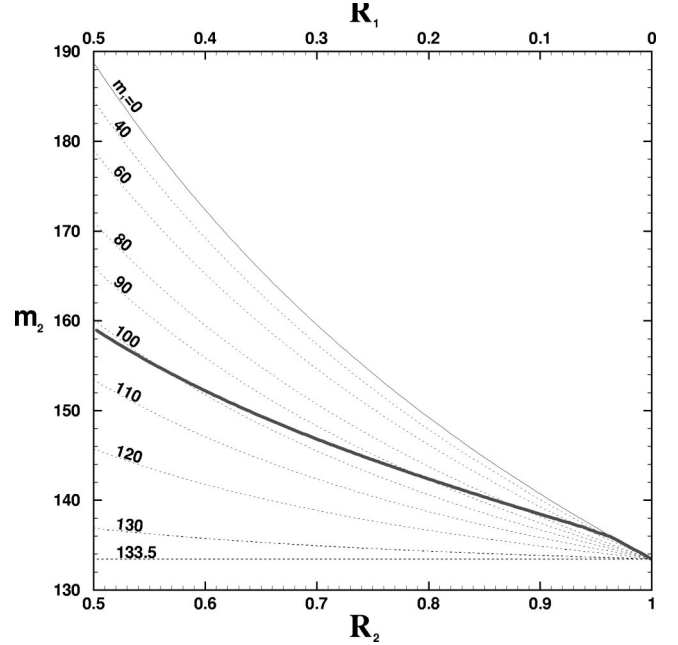


FIG. 3. Upper limits on the mass m_2 (in GeV) against R_2 , for different values of m_1 (as indicated on each line in GeV), assuming $m_t^{\text{pole}} = 173.8$ GeV and $M_{\text{SUSY}} \leq 1$ TeV. $R_1 = 1 - R_2$ is shown on the top axis. The thick straight line corresponds to LEP-II lower limits on m_1 vs R_1 .

fixed values of m_1 (as indicated on each curve). They can be used to obtain upper limits on the mass m_2 , in the regime $R_1 < 1/2$, for arbitrary experimental lower limits on the mass m_1 versus R_1 . For each value of the coupling R_1 , which would correspond to a vertical line in Fig. 3, one has to find the point where this vertical line crosses the dotted curve associated to the corresponding experimental lower limit on m_1 . Joining these points by a curve leads to the upper limit on m_2 as a function of R_2 . We have indicated as a thick straight line in Fig. 3 the present CERN e^+e^- collider LEP-II bounds [23], which give, in the “worst case” scenario, an upper limit on m_2 of ≈ 160 GeV for $R_2 \approx 1/2$.

Lower experimental limits on a Higgs boson with $R > 1/2$ restrict the allowed regime for m_2 (for $R_2 > 1/2$) in Fig. 3 from below. The present lower limits on m_2 from LEP-II are not visible in Fig. 3, since we have only shown the range $m_2 > 130$ GeV. Possibly Higgs searches at Tevatron Run-II will push the lower limits on m_2 upwards into this range. This would be necessary if one aims at an exclusion of this regime of the nMSSM. Then, lower limits on the mass m_2 —for any value of R_2 between $1/2$ and 1 —of at least 133.5 GeV are required. The precise experimental lower limits on m_2 as a function of R_2 , which would be needed to this end, will depend on the achieved lower limits on m_1 as a function of R_1 in the regime $R_1 < 1/2$.

In principle, from Eq. (26), one could have $R_2 > R_1$ with R_2 as small as $1/3$. However, in the regime $1/3 < R_2 < 1/2$, the upper bound on m_2 as a function of R_2 for different fixed values of m_1 can only be saturated if $R_1 = R_2$. It is then sufficient to look for the lightest Higgs boson S_1 (i.e., for a

Higgs boson with a coupling $1/3 < R < 1/2$ and a mass $m \lesssim 133.5$ GeV) to cover this region of the parameter space of the nMSSM.

Imposing universality of the soft terms at the GUT scale, CCB constraint [Eq. (17)] and LEP-II experimental constraints on Higgs masses [23] one finds the more restrictive bound $m_1 < 122$ GeV under the same hypotheses ($m_t^{\text{pole}} = 173.8 \pm 5.2$ GeV and $M_{\text{SUSY}} < 1$ TeV) and $m_2 < 135$ GeV in the case where S_1 is mainly singlet ($R_1 < R_2$).

One can notice that these results are the same as those obtained in the NMSSM [24]. This comes from the fact that the nonsinglet part of the CP -even mass matrix (21) as well as the singlet-nonsinglet mixing terms are the same in both models, giving the same upper limit on m_1 , Eq. (22) and Fig. 2. Even though they are not similar, the CP -even singlet mass term [the 3×3 element in Eq. (21)] is “free” in both models, i.e., it can take any value between 0 and ~ 1 TeV. This explains why the curves displayed in Fig. 3 are the same in both models, the upper bounds on m_2 steaming from degenerate singlet-nonsinglet mass terms, degeneration lifted by the mixings [off-diagonal 1×3 and 2×3 terms in Eq. (22)].

Thus, the phenomenological potential of the nMSSM is not less exciting in the Higgs boson sector than it is in the sparticle sector. On the one hand, the necessary (but not sufficient) condition for testing the complete parameter space of the nMSSM is to rule out a CP -even Higgs boson with a coupling $1/3 < R < 1$ and a mass below 135 GeV. On the other hand, the sufficient condition (i.e., the precise upper bound on m_2 versus R_2) depends on the achieved lower bound on the mass of a “weakly” coupled Higgs boson (with $0 < R < 1/2$) and can be obtained from Fig. 3. At the Tevatron Run-II this would probably require an integrated luminosity of up to 30 fb^{-1} [25]. If this cannot be achieved, one has to wait for the advent of the LHC, in order to know whether the nMSSM is actually realized in nature.

An intriguing example of possible evidence of the nMSSM at future hadron-hadron colliders is the following. Recall that a crucial signature for an intermediate Higgs boson at both the Tevatron and the CERN Large Hadron Collider (LHC) is the one produced via $q\bar{q}' \rightarrow W^\pm$ Higgs boson (with a smaller contribution from $q\bar{q} \rightarrow Z$ Higgs boson as well), with the gauge vector yielding high transverse-momentum and isolated leptons and the scalar Higgs boson decaying into $b\bar{b}$ pairs. Now, let us imagine that Higgs boson searches in this mode have finally revealed the evidence of a light scalar Higgs boson resonance, but no further Higgs boson states are detected up to well above the EW scale. (In this respect, the LHC is a better example to illustrate, as compared to the Tevatron, because of its much extended scope in mass.) This scenario could be realized in the MSSM, if the latter is in the above mentioned decoupling regime, where one has $m_{H^\pm} \approx m_H \approx m_A \gg M_Z$ and the strength of the lightest Higgs boson couplings to the gauge vector bosons W^\pm and Z approaches unity, $R_h \equiv \sin^2(\beta - \alpha) \approx 1$. Indeed, this phenomenology can also be realized in the

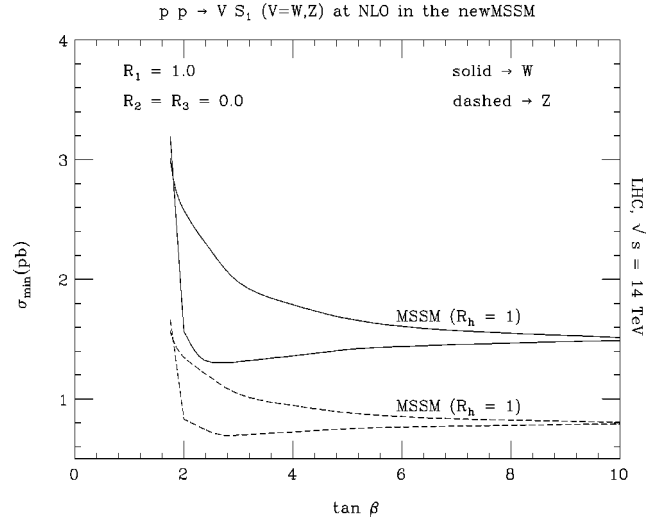


FIG. 4. Minimum of the production cross section of a neutral scalar Higgs with SM-like couplings to W^\pm and Z gauge bosons at the LHC, as a function of $\tan \beta$, in correspondence of the maximum values of its mass, in both the MSSM and the nMSSM.

nMSSM when, e.g., $R_1 = 1$ and $R_2 = 0$.² Under these circumstances, the mass of the h scalar would only depend on $\tan \beta$ and the minimum of the Higgs production cross section is obtained, in both SUSY models, in correspondence of the maximum Higgs mass. Figure 4 shows this dependence for both the MSSM and the nMSSM at the LHC, with $\sqrt{s} = 14$ TeV. (The trend is very similar at the Tevatron, where $\sqrt{s} = 2$ TeV.) Over a broad range in $\tan \beta$, the production rates of the latter are significantly below those of the former, with a minimum at $\tan \beta \approx 2.7$ (for $m_t = 173.8$ GeV), owing to the peculiar $\tan \beta$ dependence of the maximum value of m_1 , as seen in Fig. 2 (recall instead that the maximum mass of the lightest Higgs boson of the MSSM increases monotonically with $\tan \beta$ [22], hence the cross section here decreases correspondingly, for $\tan \beta \lesssim 40$).

Thus, if $\tan \beta$ has already been measured in another context (e.g., in SUSY sparticle processes), to detect an isolated scalar resonance in the $b\bar{b}$ channel (the dominant decay model in either model for $M_{\text{Higgs}} \lesssim 133.5$ GeV), at a rate well below the minimum one predicted by the MSSM, could imply that physics beyond the latter would be realized in nature. In fact, the reader should recall that the $\text{BR}(\text{Higgs boson} \rightarrow b\bar{b})$ is basically the same in both models [26] and the combined uncertainties on the rates of the latter and of the production modes (due to higher-order effects, parton distribution functions, hard scale dependence, etc.) are of the order of just a few percent [27]. In contrast, the differences between the MSSM and nMSSM rates can be as large as a factor of two, in the vicinity of $\tan \beta \approx 2.7$, and well above the mentioned uncertainties for $\tan \beta$ up to 10 or so. Besides,

²Further notice that no matter their actual mass value, pseudo-scalar neutral Higgs states of either model cannot be produced via the above two processes, as their coupling to W^\pm and Z vectors is prohibited at tree-level.

kinematical analysis of the $b\bar{b}$ system might provide further evidence in this respect, if the mass resolution is larger than the difference between the Higgs mass values, as predicted by the two models for a given $\tan\beta$. Similar arguments can be made for the case of the $gg \rightarrow \text{Higgs boson} \rightarrow \gamma\gamma$ signature too, however, the production and decay phenomenology is here much more involved (because loop processes take place at either stage), so that we leave it aside for future consideration [28].

IV. SUMMARY AND CONCLUSIONS

We have studied the phenomenology of the nMSSM, which promotes the μ -parameter into a singlet superfield, hence solving the well-known μ -problem of the MSSM. In addition, cubic self-couplings and possible dimensionful couplings are avoided in the nMSSM thanks to a global discrete R symmetry, in turn broken by supergravity-induced tadpole corrections, which solves both the so-called “domain wall” and “axion” problems. The new model is truly minimal, in the sense that—despite incorporating new fields—it can be parametrized by the same number of inputs as in the MSSM in the universal case. Its phenomenology stands out quite different from that of both the ordinary MSSM and the so-called next-to-minimal supersymmetric standard model (NMSSM)—where the cubic self-interacting coupling is instead present. In particular, the following aspects emerged as crucial from our analysis.

Assuming an accuracy up to the dominant top-stop contributions at two-loop level and depending on the (reduced) couplings of the two lighter CP -even Higgs bosons to the Z boson, we have found that the upper limit on the mass of the lightest Higgs boson state of the nMSSM can be 133.5 GeV, in correspondence of the central value of the top mass (i.e., $m_t = 173.8$ GeV), that is, about 10 GeV higher than the MSSM value and within the Tevatron Run-II reach. In addition, the upper limit on the mass of the lightest “observable” Higgs boson (i.e., the next-to-lightest one, when the lightest one couples invisibly to the Z boson) could be as high as 160 GeV but still within the LHC scope.

To remain with the Higgs sector, we also have described a benchmark example that could allow one to phenomenologically distinguish the nMSSM from the MSSM in the search for the lightest Higgs state at future hadron colliders, such as the Fermilab Tevatron Run-II and the LHC. If only one neutral Higgs state is accessible through associated production with an EW gauge vector W^\pm or Z , via its $b\bar{b}$ decays, the knowledge of $\tan\beta$ and of the production rate of such Higgs process could be enough to assign such a Higgs state to one or the other of the two models, even prior to the investigation of the mass resonance that can eventually be reconstructed from the $b\bar{b}$ system. Production and decay studies of all other Higgs states of the nMSSM are now in progress [28].

Despite the remarkable dissimilarities seen so far between the nMSSM and the MSSM, one might quite rightly question

that the phenomenology of the nMSSM is very similar to that of the NMSSM, as far as the Higgs sector is concerned. However, a dramatic difference is revealed between these two models, if one investigates CP -violation effects. In fact, no matter the actual value of λ , a peculiar feature of the nMSSM is the following: Contrary to the NMSSM case, spontaneous CP violation cannot occur in the Higgs sector of the nMSSM, neither at tree-level nor at one-loop.

Finally, further differences between the nMSSM and any other model can be appreciated in the sparticle sector. In fact, here, two concurrent aspects render the phenomenology of the former both more “spectacular” and “natural,” in comparison to the MSSM and the NMSSM, respectively. First, the lightest neutralino appears to be an almost pure singlet state. Secondly, such a state (the “singlino”) is always the LSP of the theory, with a very small mass (varying from a few MeV to a few GeV). The consequence is twofold. On the one hand, in comparison to the NMSSM, the singlino LSP scenario requires no strong constraints to be imposed on the parameter space. On the other hand, in comparison to the MSSM, any sparticle decay chain involves a further step, the NLSP \rightarrow LSP transition, giving rise to additional cascades.

The stimulating issue that such an LSP could be a good dark matter candidate or, alternatively, could be excluded from cosmological arguments, also deserves attention. However, a quantitative analysis in this respect was far beyond the intention of this paper.

Concluding, the nMSSM is, at the same time, the *best theoretically motivated* and the *most economical* SUSY extension of the SM. Here, we have pointed out differences or similarities between the new model, the NMSSM and the MSSM, in both the Higgs and neutralino sectors. Future collider experiments, at Tevatron Run-II and LHC, will be able to prove whether or not the nMSSM is the realization of Supersymmetry that nature has chosen.

Note added. While finalizing this paper, whose main results were made public already in Ref. [29], we became aware of Ref. [30], by Panagiotakopoulos and Pilaftsis, which deals with a similar subject. In this respect, although we agree with the more general statements given in this other paper, we would like to remark that there only the top-quark–top-squark one-loop corrections were used, whereas here we have calculated also the corresponding two-loop contributions. This explains the significant discrepancy between their and our upper limit on the lightest Higgs boson mass $m_1 < 150$ GeV versus $m_1 < 133.5$ GeV.

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